Roll No.							

24-MA-23

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
JUNE - JULY 2024

MATHEMATICS
Paper - III
[Topology - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 a) Prove that every closed subspace of a compact space is compact **(10 Marks)**

b) Let $f : X \rightarrow Y$ be a bijective continuous function. If X is compact space and Y is a Hausdorff space. Then prove that f is a homeomorphism. **(05 Marks)**

OR

Let X be a topological space. Then Prove that the following statements are equivalent : **(15 Marks)**

a) X is compact.

b) For each family $\{F_\alpha \mid \alpha \in \wedge\}$ of closed sets in X with $\bigcap\limits_{\alpha \in \wedge} F_\alpha = \phi$,

there is a finite subfamily $\{ F_{\alpha_i} \mid i = 1, 2, 3, \ldots, n\}$ such that $\bigcap\limits_{i=1}^{n} F_{\alpha_i} = \phi$

c) Every family of closed sets in X with FIP has a non empty inter section.

Q. 2 a) Let X be a topological space. Let one - point sets in X be closed. Then prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$ **(10 Marks)**

b) Prove that very completely normal space is a normal space. **(05 Marks)**

OR

Prove that every compact Hausdorff space is normal. **(15 Marks)**

Q. 3 a) Prove that the product of two first countable spaces is first countable. **(05 Marks)**

P.T.O.

b) Prove that the product $X = \prod_{i \in I} X_i$ is regular if each coordinate space X_i is regular. **(10 Marks)**

OR

State and Prove Tychonoff's theorem. **(15 Marks)**

Q. 4 a) Let X be a topological space and $x \in X$. Then prove that the family of all neighbourhoods of x defines a filter on X. **(05 Marks)**

b) Let X be a topological space, then prove that x is a cluster point of a filter F if and only if there exists some filter finer than F (i.e. sub filter of F) converges to x . **(10 Marks)**

OR

Prove that for a filter F on X, the following statements are equivalent - **(15 Marks)**

a) F is an ultra filter.

b) For any $A \subset X$, either $A \in F$ or $X - A \in F$

c) For any $A, B \subset X$,

$A \cup B \in F \Leftrightarrow$ either $A \in F$ or $B \in F$.

Q. 5 Let X be a topological space and $x_0 \in X$. If $C(X, x_0)$ denote the collection of all closed paths (or loops) based at x_0. then prove that the relation "homotopy modulo x_0" is an equivalence relation on $C(X, x_0)$ **(15 Marks)**

OR

a) In a simply connected space X, prove that any two paths having the same initial and final points are path homotopic. **(05 Marks)**

b) Prove that the map $p : \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map. **(10 Marks)**

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